

ArsDigitaUniversity
Month8:TheoryofComputation
ProfessorShaiSimonson

Assignment4

1.ContextFreeorNot

DetermineandprovewhethereachofthefollowinglanguagesisContextFreeornot.

- a. $\{1^k 0^i 1^j 0^k \mid i, j, k > 0\}$.
- b. $\{w \# x \mid w \text{ is a substring of } x, \text{ where } w, x \text{ are in } \{0, 1\}^*\}$.
- c. $\{0^i 1^j 0^i \mid i, j > 0\}$.
- d. **Extra Credit:** The complement of $\{(0^n 1^n)^m \mid m, n > 0\}$.

2.DecisionAlgorithms

Describealgorithmstodecide theproblemsbelow.

- a. DoesagivenDeterministicPushdownAutomatongenerate $(0+1)^*$?
- b. GivenaCFGandastringzinitlanguage,doesthestringhave2distinctderivationtrees?
(Note:youralgorithmdoesnottestwhetherornotthegrammarisambiguous!Forthatyou
wouldhavetotesteverystring.)
- c. Text:4.12:ShowthattheproblemoftestingwhetheracCFGgeneratessomestringin 1^* is
decidable.
- d. **Extra Credit:** Text4.13:ShowthattheproblemoftestingwhetheracCFGgeneratesall
stringsin 1^* isdecidable.

3.ClosureProblemsforCFL's

- a. ExplainwhytheintersectionofaregularlanguageandaCFLmustbeaCFL(i.e.CFL'sare
closedunderintersectionwithregularsets).Youshouldillustrateyourargumentby
constructingthemachine that generates $L \cap R$, where $L = 0^n 1^n$ and $R =$
 $(0+1)^* 1 10(0+1)^*$.
- b. ShowthattheintersectionofaregularlanguageandaCFLis *not*necessarilyregular(thought
mustbeaCFL -seethepreviousproblem).
- c. LetLbesomeregularsetinwhichallstringshappentohavelengthequaltoamultipleof
three.LetTwist3(L)bethesetofallstringsinLwhereeverythreesymbolsarereversed.For
exampleifL={aag,cttgta,ttggag,agc,...}thenTwist3(L)={gaa,ttcatg,gttgagcga,...}.
ExplainwhyTwist3(L)isaCFL.Youshouldillustrateyourargumentbyconstructingthemachine
thatgenerate sTwist3(L),whereL= $(0+1)^* 1 10(0+1)^*$.
- d. **Extra Credit:** IsTwist3(L)regularifLisregular?

4. ParsingandtheCYKDecisionAlgorithm

- a. ExhibitthetableyougetbydoingtheCYKalgorithmonthestings00000and000000forthe
grammarbelow.

$$\begin{array}{ll} S \rightarrow AB \mid BC & A \rightarrow BA \mid 0 \\ B \rightarrow CC \mid 1 & C \rightarrow AB \mid 0 \end{array}$$

- b. WriteaNPDA thatacceptsexactlywhatthegrammarabovegenerates.

5. ExtraCredit: Chomsky-3NormalForm

A grammar is in CNF if every production is of the form $A \rightarrow BC$ or $A \rightarrow b$.

- How many production steps does a CNF grammar use to generate a string of length n ? Explain.
- If a CNF grammar has n non-terminals, then how long does a string have to be, for it to be bound by the pumping lemma? Explain.
- Can every CNF grammar be put into CNF? Explain why or why not.

6. Turing Machine Basics

- Text 3.1a.
- Text 3.1c.
- Text 3.2a.
- Text 3.2d.

7. Turing Machine Design

- Design a TM that accepts the language of odd integers written in binary.
- Design a TM program that accepts the language $a^n b^m c^n$, where a, b, c are in $\{0,1\}^*$, and $a+b=c$, where a, b and c are interpreted as positive binary integers.
- Design a TM that enumerates the language of odd integers written in binary.
- Think about how tedious it would be to design a TM that enumerates all primes in binary.

8. Turing Recognition and Turing Decidability.

- Text 3.14a.
- Text 3.14d.
- Text 3.15a.
- Text 3.15c.

9. ExtraCredit: A Problem that is Easier than it Seems

Show that if every subset of a set is a CFL, then the set must be regular.